

Math (Science)	Group-I	Paper
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any Six (6) questions: 12

(i) Define identity matrix.

Ans A diagonal matrix is called identity (unit) matrix, if all diagonal entries are 1. It is denoted by I.

e.g., $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a 3-by-3 identity matrix.

(ii) Find the product of $[x \ y] \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

Ans

$$\begin{aligned}
 &= [x \ y] \begin{bmatrix} 2 \\ -1 \end{bmatrix} \\
 &= [(x)(2) + (y)(-1)] \\
 &= [2x - y]
 \end{aligned}$$

(iii) Define irrational numbers.

Ans The numbers which cannot be expressed as a quotient of integers are called irrational numbers.

The set of irrational numbers is denoted by Q' . The numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π and e are all irrational numbers.

(iv) Simplify: $4\sqrt{32}$

Ans

$$\begin{aligned}
 4\sqrt{32} &= 4\sqrt{2^5} \\
 &= 4\sqrt{2^4 \times 2} \\
 &= (2^4 \times 2)^{1/4} \\
 &= 2^{4 \times \frac{1}{4}} \times 2^{\frac{1}{4}} \\
 &= 2 \times 2^{\frac{1}{4}} \\
 &= 2\sqrt[4]{2}
 \end{aligned}$$

(v) Define scientific notation.

Ans A number written in the form $a \times 10^n$, where $1 \leq a \leq 10$ and n is an integer, then it is called the scientific notation.

(vi) If $A = \pi r^2$, find A , when $\pi = \frac{22}{7}$ and $r = 15$.

Ans

$$A = \pi r^2$$

$$A = \frac{22}{7} (15)^2$$

Taking log both side:

$$\log A = \log \left(\frac{22}{7} (15)^2 \right)$$

$$= \log 22 - \log 7 + 2 \log 15$$

$$= 1.3424 - 0.8451 + 2(1.1761)$$

$$= 1.3424 - 0.8451 + 2.3522$$

$$= 2.8494$$

Taking anti-log:

$$A = \text{Antilog}(2.8494)$$

$$A = 707.1$$

(vii) If $a + b = 5$ and $a - b = \sqrt{17}$, then find the value of ab .

Ans

$$a + b = 5$$

$$a - b = \sqrt{17}$$

$$ab = ?$$

We know that

$$(a + b)^2 - (a - b)^2 = 4ab$$

Putting values of $a + b$ and $a - b$.

$$(5)^2 - (\sqrt{17})^2 = 4ab$$

$$25 - 17 = 4ab$$

$$8 = 4ab$$

$$\frac{8}{4} = ab$$

$$\boxed{ab = 2}$$

(viii) Factorize: $64x^3 + 343y^3$

Ans $64x^3 + 343y^3 = (4x)^3 + (7y)^3$

$$= (4x + 7y)[(4x)^2 - (4x)(7y) + (7y)^2]$$

$$= (4x + 7y)(16x^2 - 28xy + 49y^2)$$

(ix) Find the remainder when $9x^2 - 6x + 2$ is divided by $x - 3$.

Ans Let,

$$P(x) = 9x^2 - 6x + 2$$

As $P(x)$ is divided by $x - 3$, then

By Remainder Theorem,

$$P(3) = 9(3)^2 - 6(3) + 2$$

$$= 81 - 18 + 2$$

$$= 65$$

3. Write short answers to any Six (6) questions: 12

(i) Define L.C.M.

Ans If an algebraic expression $p(x)$ is exactly divisible by two or more expressions, then $p(x)$ is called Common Multiple of the given expressions. The Least Common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.

(ii) Solve for x : $|3x - 5| = 4$

Ans

$$3x - 5 = 4 \quad ; \quad 3x - 5 = -4$$

$$3x = 4 + 5 \quad ; \quad 3x = -4 + 5$$

$$3x = 9; \quad 3x = 1$$

$$x = \frac{9}{3}; \quad x = \frac{1}{3}$$

$$x = 3$$

(iii) Solve the equation: $\frac{3x}{2} - \frac{x-2}{2} = \frac{25}{6}$

Ans

$$\frac{3x - x + 2}{2} = \frac{25}{6}$$

$$\frac{2x + 2}{2} = \frac{25}{6}$$

$$\frac{2(x + 1)}{2} = \frac{25}{6}$$

$$6(x + 1) = 25$$

$$6x + 6 = 25$$

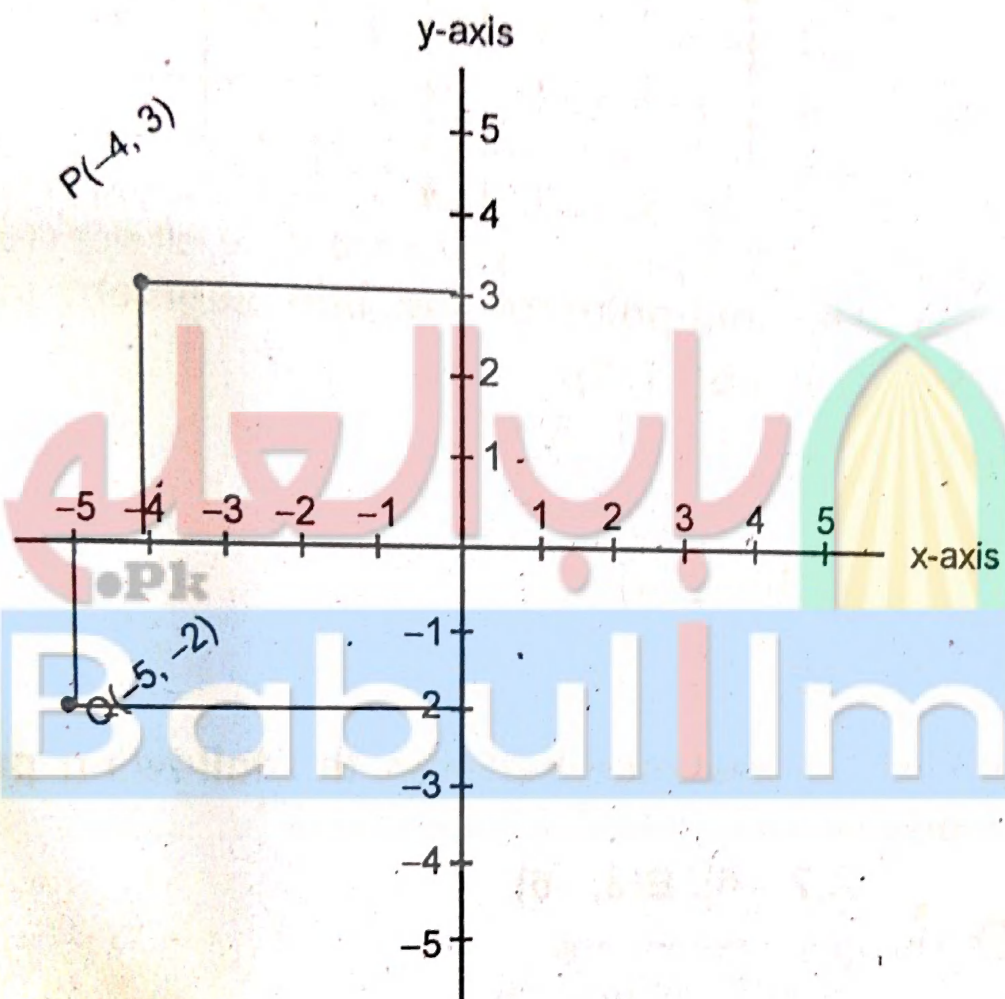
$$6x = 25 - 6$$

$$6x = 19$$

$$x = \frac{19}{6}$$

- (iv) Determine the quadrant of the coordinate plane in which the points $P(-4, 3)$ and $Q(-5, -2)$ lie.

Ans



$P(-4, 3)$ point is present in II quadrant.

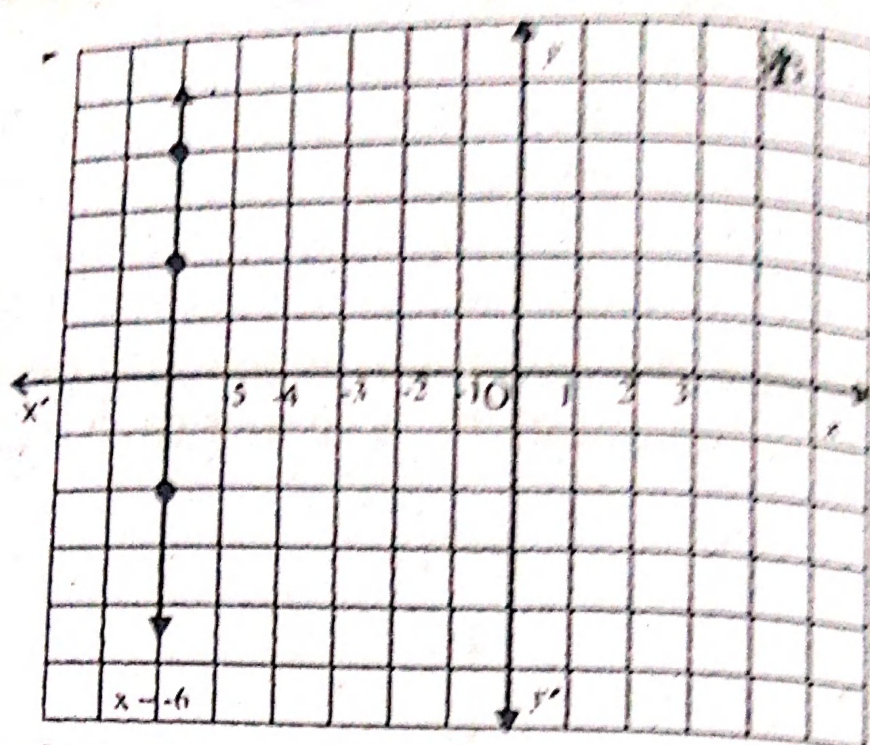
$Q(-5, -2)$ point is present in III quadrant.

- (v) Draw the graph of the following $x = -6$ (graph is page size).

Ans The table value for x and y :

$x =$	-6	-6	-6	-6	-6	-6
$y =$	2	4	5	-1	-2

Graph:



Graph of $x = -6$ is parallel to y -axis and at the left side of it.

- (vi) Find the mid-point of the line segment joining $A(2, 5)$ and $B(-1, 1)$.

Ans

$$M = \left(\frac{2 + (-1)}{2}, \frac{5 + 1}{2} \right)$$

$$M = \left(\frac{1}{2}, 3 \right)$$

$$M = \left(\frac{1}{2}, 3 \right)$$

- (vii) Find the distance between the following pair of points:

$A(2, -6)$, $B(3, -6)$

Ans

The given points are:

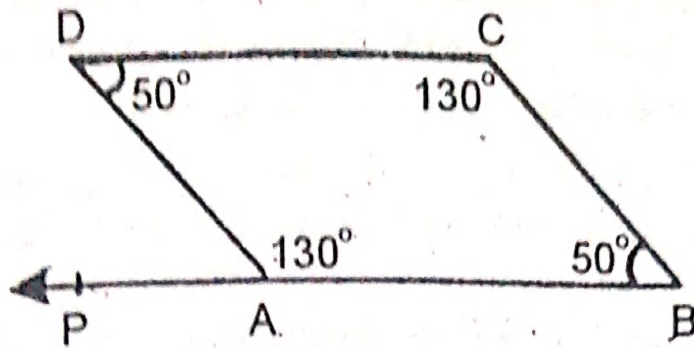
$A(2, -6)$, $B(3, -6)$

The distance formula is:

$$\begin{aligned} d = |AB| &= \sqrt{(3 - 2)^2 + (-6 + 6)^2} \\ &= \sqrt{(1)^2 + (0)^2} \\ &= 1 \end{aligned}$$

- (viii) One angle of a parallelogram is 130° . Find the measures of its remaining angles.

Ans

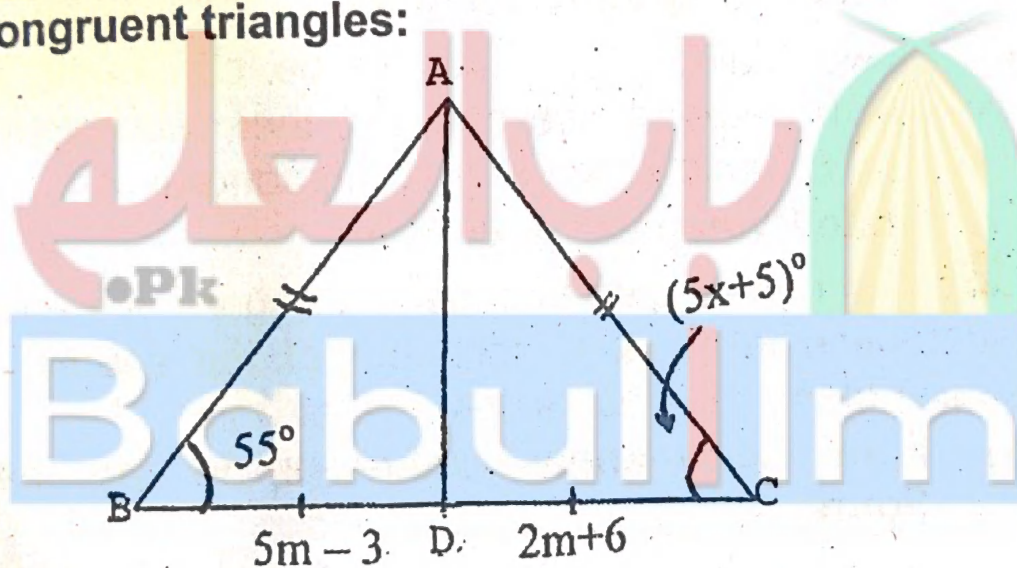


$$\begin{aligned}\angle B &\cong \angle C \\ m\angle A &= 130^\circ \\ m\angle C &= 130^\circ \\ m\angle B &= 180^\circ - m\angle A \\ &= 180^\circ - 130^\circ = 50^\circ\end{aligned}$$

As

$$\begin{aligned}\angle B &= \angle D \\ m\angle C &= 50^\circ\end{aligned}$$

(ix) Find the value of unknown for the given congruent triangles:



Ans

$$\begin{aligned}\text{(i)} \quad (5x + 5)^\circ &= 55^\circ \\ 5x + 5 &= 55 \\ 5x &= 55 - 5 \\ 5x &= 50 \\ \boxed{x} &= \boxed{10}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad 5m - 3 &= 2m + 6 \\ 5m - 2m &= 6 + 3 \\ 3m &= 9 \\ \boxed{m} &= \boxed{3}\end{aligned}$$

4. Write short answers to any Six (6) questions: 12

(i) Define bisector of a line segment.

Ans A line l is called a right bisector of a line segment, if l is perpendicular to the line segment and passes through its mid-point.

(ii) If 3 cm and 4 cm are lengths of two sides of right angle triangle, then what should be the third length of the triangle?

Ans

$$\begin{aligned}(\text{Hypotenuse})^2 &= (\text{Perpendicular})^2 + (\text{Base})^2 \\(\text{Hypotenuse})^2 &= (3)^2 + (4)^2 \\(\text{Hypotenuse})^2 &= 9 + 16 \\(\text{Hypotenuse})^2 &= 25 \\\sqrt{(\text{Hypotenuse})^2} &= \sqrt{25} \\(\text{Hypotenuse}) &= 5 \text{ cm}\end{aligned}$$

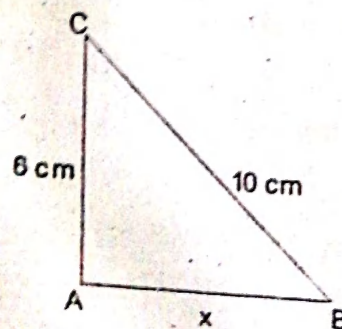
(iii) Define congruent triangles.

Ans Two triangles are said to be congruent, if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

(iv) State Pythagoras theorem.

Ans In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

(v) Find the unknown value (x) in the following figure:



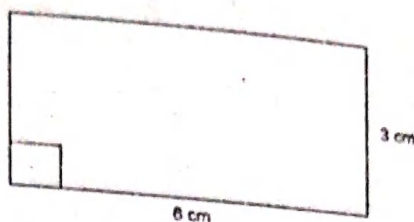
Ans As

$$\begin{aligned}(\text{Hyp})^2 &= (\text{Base})^2 + (\text{Perpendicular})^2 \\(10)^2 &= (x)^2 + (6)^2 \\100 &= x^2 + 36 \\100 - 36 &= x^2 \\x^2 &= 64 \\x &= 8 \text{ cm}\end{aligned}$$

(vi) Define triangular region.

Ans A triangular region is the union of a triangle and its interior, i.e., the three line segments forming the triangle and its interior.

(vii) Find area of:



Ans Length of rectangle = 6 cm
Width of // // = 3 cm
Area of // // = 6×3
 $= 18 \text{ cm}^2$

(viii) Define orthocentre of a triangle.

Ans The point of concurrency of the three altitudes of a Δ is called its orthocentre.

(ix) Define circumcentre of a triangle.

Ans The point of concurrency of the three perpendicular bisectors of the sides of a triangle is called circumcentre of a triangle.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve by the matrix inversion method: (4)

$$2x + y = 3$$

$$6x + 5y = 1$$

Ans

$$2x + y = 3 \quad (i)$$

$$6x + 5y = 1 \quad (ii)$$

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$AX = C$$

$$X = A^{-1}C$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 10 - 6 = 4$$

$$\text{Adj}(A) = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$A^{-1} \frac{\text{Adj}(A)}{|A|} = \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$X = \frac{1}{4} \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix} = \begin{bmatrix} \frac{14}{4} \\ \frac{-16}{4} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

Hence,

$$x = \frac{7}{2}, y = -4$$

(b) Show that: $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Ans $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

L.H.S

$$\begin{aligned} &= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \\ &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\ &= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \\ &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\ &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= x^0 \\ &= 1 = \text{R.H.S} \end{aligned}$$

Q.6.(a) Use log table to find the value of:

$$\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

Ans For Answer see Paper 2017 (Group-I), Q.6.(a).

(b) If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$, then find the value of $xy + yz + zx$.

Ans

$$\begin{aligned} x + y + z &= 12 \\ x^2 + y^2 + z^2 &= 64 \\ xy + yz + zx &= ? \\ (x + y + z)^2 &= x^2 + y^2 + z^2 + 2(xy + yz + zx) \\ (12)^2 &= 64 + 2(xy + yz + zx) \\ 144 &= 64 + 2(xy + yz + zx) \\ 144 - 64 &= 2(xy + yz + zx) \end{aligned}$$

$$80 = 2(xy + yz + zx)$$

$$\frac{80}{2} = (xy + yz + zx)$$

$$\boxed{xy + yz + zx = 40}$$

Q.7.(a) Factorize: $(x^2 - 4x)(x^2 - 4x - 1) - 20$ (4)

Ans Let $x^2 - 4x = y$

$$\begin{aligned}
 &= (x^2 - 4x)(x^2 - 4x - 1) - 20 \\
 &= (y)(y - 1) - 20 \\
 &= y^2 - y - 20 \\
 &= y^2 - 5y + 4y - 20 \\
 &= y(y - 5) + 4(y - 5) \\
 &= (y - 5)(y + 4)
 \end{aligned}$$

Putting $y = x^2 - 4x$

$$\begin{aligned}
 &= (x^2 - 4x - 5)(x^2 - 4x + 4) \\
 &= [x^2 - 5x + x - 5][x^2 - 2x - 2x + 4] \\
 &= [x(x - 5) + 1(x - 5)][x(x - 2) - 2(x - 2)] \\
 &= [(x + 1)(x - 5)][(x - 2)(x - 2)] \\
 &= \boxed{(x - 5)(x + 1)(x - 2)^2}
 \end{aligned}$$

(b) Use division method to find the square root of the expression: (4)

$$4x^4 + 12x^3 + x^2 - 12x + 4$$

Ans

$$\begin{array}{r}
 2x^2 + 3x - 2 \\
 \hline
 2x^2 \overline{) 4x^4 + 12x^3 + x^2 - 12x + 4} \\
 \underline{\pm 4x^4} \\
 4x^2 + 3x \overline{) 12x^3 + x^2 - 12x + 4} \\
 \underline{\pm 12x^3 \pm 9x^2} \\
 4x^2 + 6x - 2 \overline{) - 8x^2 - 12x + 4} \\
 \underline{\mp 8x^2 \mp 12x \pm 4} \\
 0
 \end{array}$$

Thus square root of given expression is $\pm (2x^2 + 3x - 2)$.

Q.8.(a) Solve the equation:

$$\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}, \left(x \neq -\frac{5}{2}\right).$$

Ans

$$\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$$

$$\frac{2x}{2x+5} + \frac{5}{4x+10} = \frac{2}{3}$$

$$\frac{2x}{2x+5} + \frac{5}{2(2x+5)} = \frac{2}{3}$$

$$\frac{2(2x) + 5}{2(2x+5)} = \frac{2}{3}$$

$$\frac{4x+5}{2(2x+5)} = \frac{2}{3}$$

By cross multiplication

$$3(4x+5) = 2[2(2x+5)]$$

$$12x+15 = 2(4x+10)$$

$$12x+15 = 8x+20$$

$$12x-8x = 20-15$$

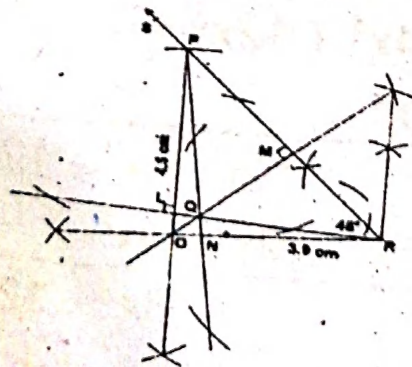
$$4x = 5$$

$$x = \frac{5}{4}$$

(b) Construct $\triangle PQR$ and draw its altitudes:

$m\overline{PQ} = 4.5$ cm, $m\overline{QR} = 3.9$ cm and $m\angle R = 45^\circ$

Ans



Steps of Construction:

1. Take $m\overline{QR}$ line = 3.9 cm.
2. At point R make angle $m\angle = 45^\circ$. With Q as centre draw an arc of radius 4.5 cm which intersect RS at the point P. Join P to Q to complete $\triangle PQR$.

3. From the vertex P drop $PL \perp QR$.
4. From the vertex Q drop $QM \perp PR$. These two altitudes meet in the point O inside the $\triangle PQR$.
5. Now from the third vertex E, drop $RN \perp PQ$.
6. We observe that this third altitude also passes through the point of intersection O of the first two altitudes.
7. Hence the three altitudes of $\triangle PQR$ are concurrent at O.

Q.9. Any point on the right bisector of a line segment is equidistant from its end point. (8)

Ans For Answer see Paper 2014 (Group-II), Q.9.

OR

Triangles on equal bases and of equal altitudes are equal in area.

Ans For Answer see Paper 2014 (Group-II), Q.9(OR).